Math 4
Final Exam Review

Name Key
Date________

Prove each.

1. $\sin x \tan x + \cos x = \sec x$

$\frac{\sin x}{1} \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{1} =$

$\frac{\sin^2 x}{\cos x} + \frac{\cos x}{1} \cdot \left(\frac{\cos x}{\cos x}\right) =$

$\frac{\sin^2 x + \cos^2 x}{\cos x}$

$\frac{1}{\cos x} =$

$\underline{\sec x} = \underline{\sec x}$

2. $\dfrac{\sin x + \cot x}{\cos x} = \tan x + \csc x$

$\frac{\sin x}{\cos x} + \frac{\cot x}{\cos x} =$

$\tan x + \frac{\frac{\cos x}{\sin x}}{\cos x} =$

$\tan x + \frac{1}{\sin x} =$

$\tan x + \csc x = \tan x + \csc x$

Solve.

3. $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0$ | $\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$ | $\sin x = 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ | $x = \frac{\pi}{2}$

4. $1 - 2\sin^2 x + 5\cos x = 2$

$-2(1-\cos^2 x) + 5\cos x - 1 = 0$

$-2 + 2\cos^2 x + 5\cos x - 1 = 0$

$2\cos^2 x + 5\cos x - 3 = 0$

$(2\cos x - 1)(\cos x + 3) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = -3$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ No solution

Simplify.

5. $\cos\left(\frac{18\pi}{12}\right)$

$\cos\left(\frac{3\pi}{2}\right)$

0

or as an angle addition

$\cos\left(\frac{12\pi}{12} + \frac{6\pi}{12}\right) = \cos\left(\pi + \frac{\pi}{2}\right)$

$= \cos\pi \cdot \cos\frac{\pi}{2} - \sin\pi \sin\frac{\pi}{2}$

$= -1 \cdot 0 - 0 \cdot 1$

$= 0$ (but why would you do it that way?!)

6. $\dfrac{\tan(85^\circ) - \tan(40^\circ)}{1 + \tan(85^\circ) \cdot \tan(40^\circ)}$

$\tan(85 - 40)$

$\tan 45^\circ$

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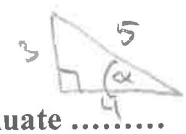
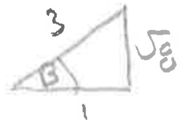
Q4

Let $\sin\alpha = -\frac{3}{5}$ with $\frac{3\pi}{2} < \alpha < 2\pi$ and $\cos\beta = \frac{1}{3}$ with β in quadrant I.



$\sin\alpha = -\frac{3}{5}$

$\cos\alpha = \frac{4}{5}$



$\cos\beta = \frac{1}{3}$

$\sin\beta = \frac{\sqrt{8}}{3}$

Evaluate ………

7. $\cos(\alpha+\beta)$

$\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$

$\frac{4}{5} \cdot \frac{1}{3} - -\frac{3}{5} \cdot \frac{\sqrt{8}}{3}$

$\frac{4+3\sqrt{8}}{15}$

8. $\tan\beta$

$\tan\beta = \frac{\sqrt{8}}{1}$

9. $\sin(\alpha+\beta)$

$\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$

$-\frac{3}{5} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{\sqrt{8}}{3}$

$\frac{-3+4\sqrt{8}}{15}$

10. $\sec\alpha$

$\frac{1}{\cos\alpha}$

$\frac{1}{\frac{4}{5}} = \frac{5}{4}$

11. In which quadrant is $\alpha+\beta$?

QI, both $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ are positive

12. Given $f(x) = x^2 - 3x + 5$, find the expression for $f'(x)$ by definition.

$f'(x) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 3(x+\Delta x) + 5 - (x^2 - 3x + 5)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + (\Delta x)^2 - \cancel{3x} - 3\Delta x + \cancel{5} - \cancel{x^2} + \cancel{3x} - \cancel{5}}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2x \cdot \Delta x + (\Delta x)^2 - 3\Delta x}{\Delta x}$

$= \lim_{\Delta x \to 0} 2x + \Delta x - 3$

$f'(x) = 2x - 3$

13. Find the average rate of change in the function $g(x) = 2x^2 - 3x + 1$ over the interval $-1 \le x \le 4$.

$\text{ARoC} = \frac{g(4) - g(-1)}{4-(-1)} \Rightarrow \frac{21-6}{5} \Rightarrow \frac{15}{5} = 3$

14. Find the equation of the line tangent to the curve $y = x^3 + 2x^2 - 5x - 1$ at $x = 2$.

$f'(x) = 3x^2 + 4x - 5$

$f(2) = 5$

$f'(2) = 15$

$y - 5 = 15(x-2)$

or $y = 15x - 25$

15. Find the derivative, $\frac{dy}{dx}$, for each:

a. $y = 2x^{-3} - 9x + 5$

$y' = -6x^{-4} - 9$

b. $y = (2x-5)(x^2+5x+3)$

$y' = (2x-5)(2x+5) + 2(x^2+5x+3)$

$= 4x^2 - 25 + 2x^2 + 10x + 6$

$= 6x^2 + 10x - 19$

c. $y = \dfrac{3x^2}{x^2 - 5x + 3}$

$y' = \dfrac{6x(x^2-5x+3) - 3x^2(2x-5)}{(x^2-5x+3)^2}$

$= \dfrac{6x^3 - 30x^2 + 18x - 6x^3 + 15x^2}{()^2}$

$= \dfrac{-15x^2 + 18x}{(x^2-5x+3)^2}$

d. $y = \sqrt{4x-7}$

$y = (4x-7)^{\frac{1}{2}}$

$y' = \frac{1}{2}(4x-7)^{-1/2} \cdot 4$

$= \dfrac{2}{\sqrt{4x-7}}$

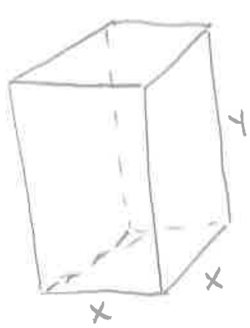
e. $y = (4x^2 - 9x)^6$

$y' = 6(4x^2-9x)^5(8x-9)$

f. $y = 4^{\pi}$

$y' = 0$

16. An open rectangular box with square base is to be made from 48 ft^2 of material. What dimensions will result in a box with the largest possible volume?



$48 = x^2 + 4xy$

$y = \dfrac{48 - x^2}{4x}$

$V = x^2 y$

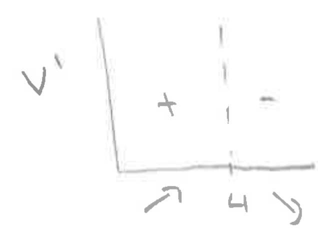
$V = x^2\left(\dfrac{48 - x^2}{4x}\right)$

$V = 12x - \frac{1}{4}x^3$

$V' = 12 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 12$

$x^2 = 16 \quad x = 4$



Max Volume when $x = 4$ ft

$y = 2$ ft

17. Consider a rectangle of perimeter 12 inches. Form a cylinder by revolving this rectangle about one of its edges. What dimensions of the rectangle will result in a cylinder of maximum volume?

$V = \pi r^2 h$

$V = \pi r^2 (6 - r)$

$V = 6\pi r^2 - \pi r^3$

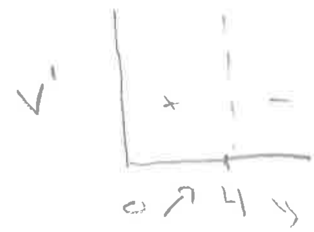
$V' = 12\pi r - 3\pi r^2$

$= 3\pi r(4 - r)$

$2h + 2r = 12$

$h + r = 6$

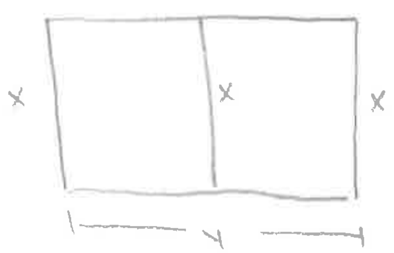
$h = 6 - r$



Max Volume when $r = 4$ in

$h = 2$ in

18. A rancher wants to construct two identical rectangular corrals using 200 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?



$A = x \cdot y$

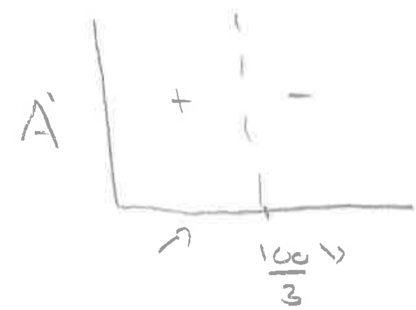
$A = x\left(100 - \frac{3}{2}x\right)$

$A = 100x - \frac{3}{2}x^2$

$A' = 100 - 3x$

$3x + 2y = 200$

$y = 100 - \frac{3}{2}x$



Max area when

$x = 33\frac{1}{3}$ ft

$y = 50$ ft

19. A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $p(t) = \frac{1}{3}t^3 - 3t^2 + 8t$. $v(t) = t^2 - 6t + 8$

a) Show that at time $t = 0$ the particle is moving to the right.
b) Find all values of t for which the particle is moving to the left.
c) What is the position of the particle at time $t = 3$?
d) When $t = 3$, what is the total distance the particle has traveled?

a) $v(0) = 8 > 0$

$\therefore$ Moving right

b) $v(t) = (t-2)(t-4)$

Moving left $2 < t < 4$

c) $P(3) = 6$

d) $P(0) = 0$

$P(2) = 6\frac{2}{3}$

$P(3) = 6$

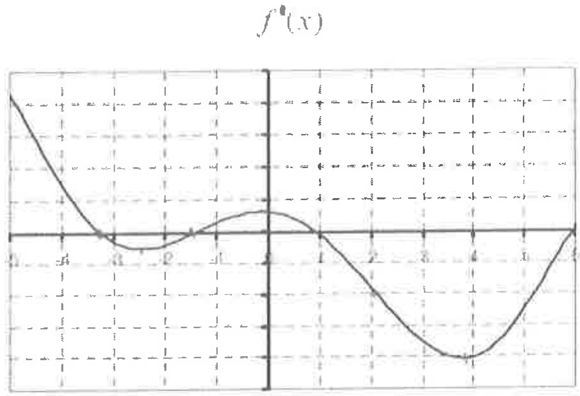
$6\frac{2}{3}$

$\frac{2}{3}$

total distance

$7\frac{1}{3}$

20. A graph of $f'(x)$ is given at the right.



A. On what interval(s) is $f(x)$ increasing? Decreasing? Explain.

Increasing, $f'(x) > 0$

$x < -3.3$

and $-1.5 < x < 1$

Decreasing when $f'(x) < 0$

$-3.3 < x < 1.5$ and

$1 < x < 6$

B. On what interval(s) is $f'(x)$ increasing? Decreasing? Explain.

Increasing on

$-2.5 < x < 0$ and $3.9 < x < 6$

Decreasing

$x < -2.5$ and $0 < x < 3.9$

C. On what interval(s) is $f(x)$ concave up? Concave down? Explain.

Same as Part B

The second derivative of $f(x)$ is the 1st derivative of $f'(x)$.

D. On what interval(s) is $f'(x)$ concave up? Concave down? Explain.

Concave up

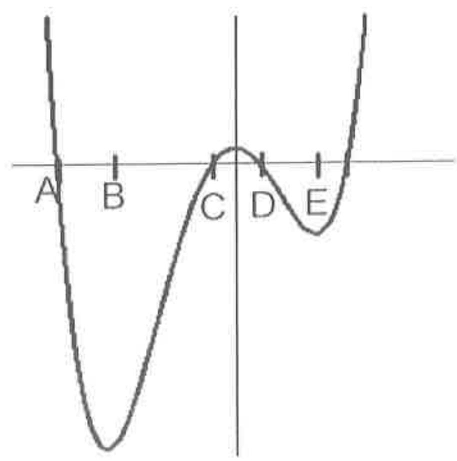
$x < -1.5$ and

$x > 2$

Concave down

$-1.5 < x < 2$

21. At the right is a graph of $f(x)$. Give the interval(s) or point(s) where $f'(x)$ is negative, positive and zero.



Negative: $x < B$

$f(x)$ decreasing

$0 < x < E$

Positive: $B < x < 0$

$f(x)$ increasing

$x > E$

Zero: $x = B$

$f(x)$ has max/min

$x = 0$

$x = E$

22. A particle moves on the x-axis (in units) such that its position at time t (in seconds) is given by the function:

$$s(t) = t^3 - 9t^2 + 15t, \quad 0 \le t \le 6$$

a. Determine the velocity & acceleration of the particle at time t.

$v(t) = 3t^2 - 18t + 15$

$a(t) = 6t - 18$

b. For what values of t is the particle at rest?

$v(t) = 0$

$3(t^2 - 6t + 5) = 0$

$3(t-5)(t-1) = 0$

$t = 1$

$t = 5$

c. For what values of t is the particle moving to the right? To the left?

$t-1$	$-$	$+$	$+$
$t-5$	$-$	$-$	$+$
	$\oplus$	$\ominus$	$\oplus$

1 5

right: $0 \le t < 1$

$5 < t \le 6$

left: $1 < t < 5$

d. What is the total distance it has traveled after 6 seconds?

$s(0) = 0$
> 7
$s(1) = 7$
> 32
$s(5) = -25$
> 7
$s(6) = -18$

total distance.

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e. What is the velocity when the acceleration is zero? Explain what your answer means in context.

$a(t) = 0$

$6t - 18 = 0$

$t = 3$

$v(3) = -12$

moving left

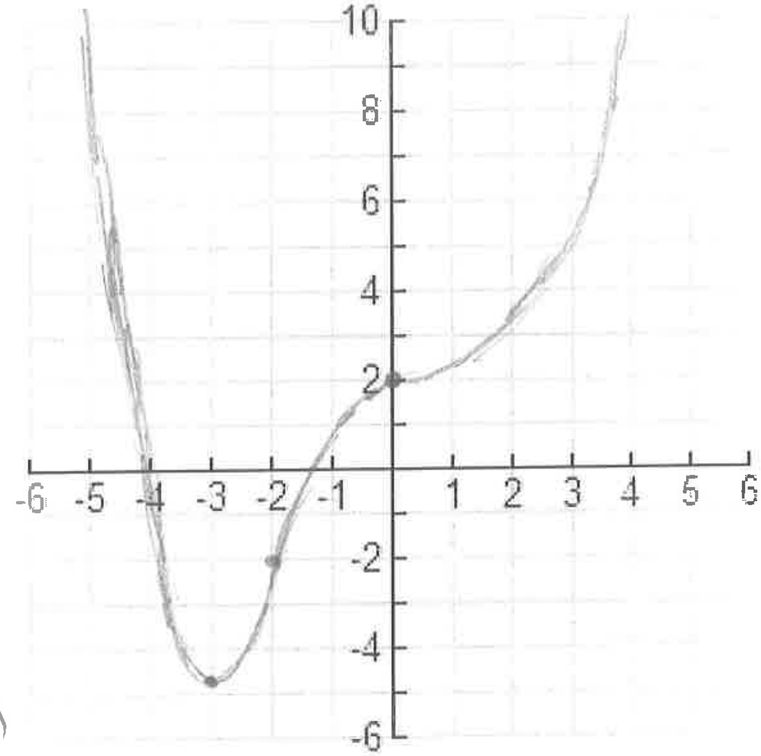
f. Is the particle speeding up or slowing down when $t = 4$ seconds?

$v(4) = -9$

$a(4) = 6$

since the velocity and acceleration at $t=4$ have different signs, the Particle is slowing down

23. Consider the equation $f(x)=\frac{1}{4}x^4+x^3+2$



a. Determine the points where there are maximums, minimums, or "flat spots".

$f'(x) = x^3 + 3x^2$

$= x^2(x+3)$

x^2	+		+		+
$x+3$	−		+		+
	↘	−3	↗	0	↗

b. Find the coordinates of the maximums, minimums, and "flat spots"

minimum at $(-3, f(-3))$

$(-3, -4.75)$

"flat spot" at $(0, f(0)) = (0, 2)$

c. Determine the concavity of $f(x)$. Your answer should be intervals.

$f''(x) = 3x^2 + 6x$

$= 3x(x+2)$

$3x$	−		−		+
$x+2$	−		+		+
	+	−2	−	0	+

Concave up:

$(-\infty, -2)$ and $(0, \infty)$

Concave down

$(-2, 0)$

d. Find the coordinates of the inflection point(s) of $f(x)$.

$(-2, f(-2)) = (-2, -2)$

$(0, f(0)) = (0, 2)$

e. Sketch the graph of $f(x)$ based on the above information. Label all points you found above. DO NOT USE YOUR CALCULATOR!! ok, then. Relax

Math 4 – Final Exam Formulas

Pythagorean Identities

$$\cos^2 x + \sin^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

Sum and Difference Identities

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Double Angle Identities

$$\sin(2x) = 2\sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\cos(2x) = 2\cos^2 x - 1$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x}$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)}$$